Galactic Dynamics with Gaia

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Galaxies

Collections of (many) stars (and gas)…

No clear unique definition in the literature, but one common one is that the stellar dynamics in these objects is ***collisionless***

The relaxation time (for a star's velocity to change by of order itself through encounters) is larger than the age of the Universe (rises with $N/\ln(N)$ and t_{cross})

Galaxies

Consider the phase-space probability distribution function P_N of a system of $N \sim 10^{11}$ equal masses μ

$$\underbrace{\frac{\partial P_N}{\partial t} + \sum_{i=1}^{N} \left[\boldsymbol{v}_i \cdot \frac{\partial P_N}{\partial \boldsymbol{x}_i} + \mu \boldsymbol{\mathcal{F}}_i^{\mathrm{tot}} \cdot \frac{\partial P_N}{\partial \boldsymbol{v}_i} \right] = 0}_{\text{Liouville}}$$

Define :

$$P_n(\Gamma_1, ..., \Gamma_n, t) = \int \mathrm{d}\Gamma_{n+1} ... \mathrm{d}\Gamma_N \, P_N(\Gamma_1, ..., \Gamma_N, t)$$

Integrate Liouville to get the BBGKY hierarchy

Galaxies

Define reduced distribution functions
(such that f_1 is the one-particle phase-space density in terms of mass):

$$f_n(\Gamma_1, ..., \Gamma_n, t) = \mu^n \frac{N!}{(N-n)!} P_n(\Gamma_1, ..., \Gamma_n, t)$$

Define g_2 such that :

$$f_2(\Gamma_1, \Gamma_2) = f_1(\Gamma_1) f_1(\Gamma_2) + g_2(\Gamma_1, \Gamma_2)$$

$$\frac{\partial f_1}{\partial t} + \boldsymbol{v}_1 \cdot \frac{\partial f_1}{\partial \boldsymbol{x}_1} + \underbrace{\left[\int d\Gamma_2\, \boldsymbol{\mathcal{F}}_{12} f_1(\Gamma_2)\right]}_{\text{-grad}(\Phi)} \cdot \frac{\partial f_1}{\partial \boldsymbol{v}_1} + \cancel{\int d\Gamma_2\, \boldsymbol{\mathcal{F}}_{12} \cdot \frac{\partial g_2(\Gamma_1, \Gamma_2)}{\partial \boldsymbol{v}_1}} = 0$$

(becomes negligible in the large N limit)

=> df_1/dt = 0 **for the « 1-star distribution function »**

The collisionless Bolzmann equation or Vlasov equation
as a fundamental equation of stellar dynamics !

« Vlasov-Poisson » system of equations

$$
\begin{cases}
\mathrm{d}f/\mathrm{dt} = 0 \Leftrightarrow \dfrac{\partial f}{\partial t} + [f, H] = \dfrac{\partial f}{\partial t} + \mathbf{v} \cdot \dfrac{\partial f}{\partial \mathbf{x}} - \dfrac{\partial \Phi}{\partial \mathbf{x}} \cdot \dfrac{\partial f}{\partial \mathbf{v}} = 0, \\
\nabla^2 \Phi = 4\pi G \displaystyle\int \mathrm{d}^3\mathbf{v}\, f
\end{cases}
$$

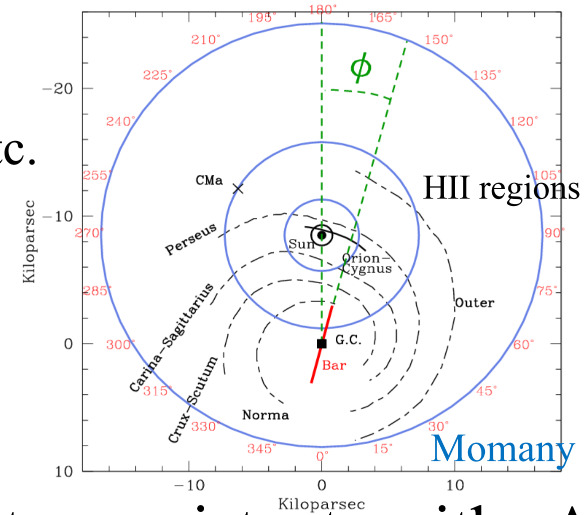
f for each of the stellar components and in principle also the dark matter component, also constrained (in configuration space) through Φ

Gaia: what do we want to know?

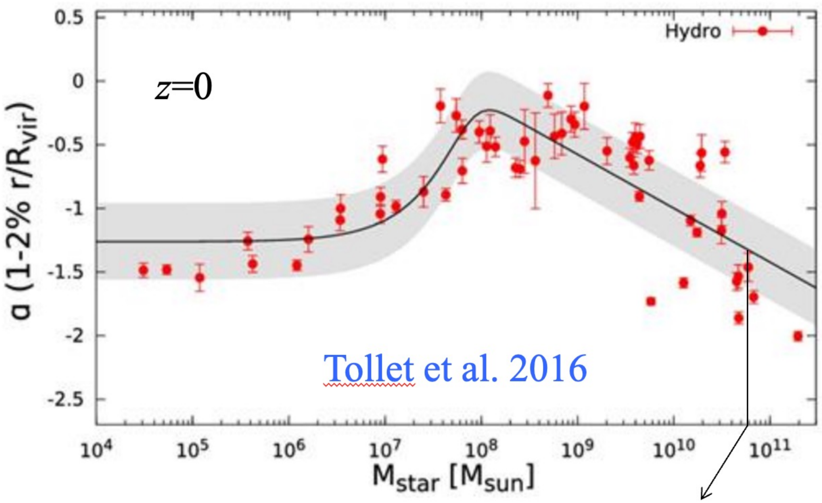
- *First type* of question: what is the **current structure** of the Galaxy?

Phase space DF for different populations

Bar structure, pattern speed, spiral arms, etc.



+ its **dark matter** distribution, and is it consistent with ΛCDM expectations ?



Gaia: what do we want to know?

- *Second type* of question: **how did the Milky Way form and evolve?**

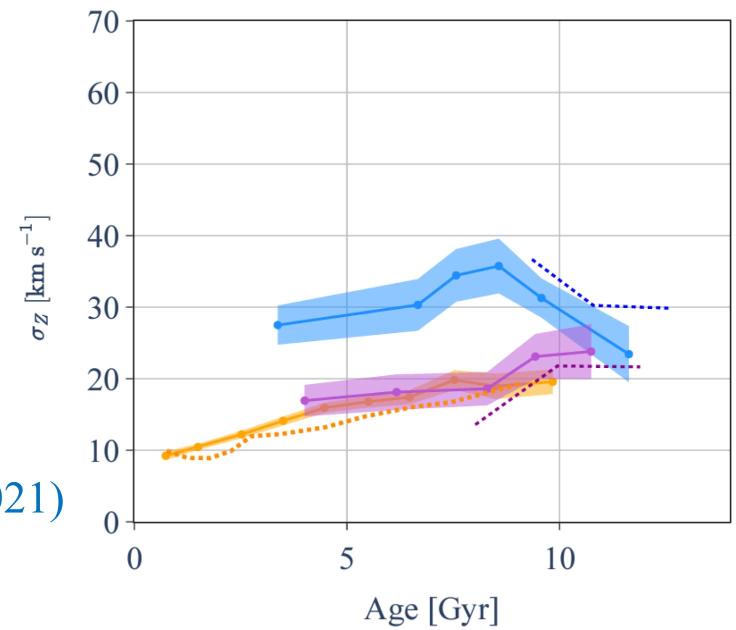
- Role of internal secular evolution and external perturbers?

- How much radial migration do the bar and spirals induce? Did the bar slow down and induce a different type of migration? How did the thin and thick disks form and evolve?

- How many past mergers etc. ?

All related to the first type of question…

Lagarde et al. (2021)



The best we know the stars (from their distances to their ages), the best answers we can get !

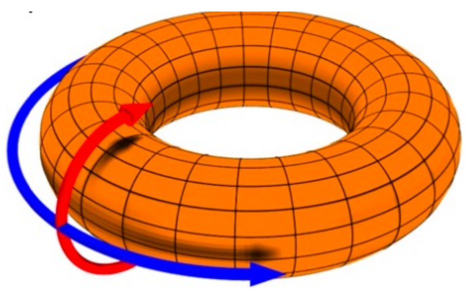
Jeans theorem

- If integrable system: $df/dt = 0 \Leftrightarrow f(I_1, I_2, I_3)$
- In axisymmetry and equilibrium: $f_0\ (E, L_z, I_3)$

- How to choose your integrals? Search for generalized momenta **J** that are integrals of motion such that H=H(**J**)

$$\dot{J}_i = -\frac{\partial H}{\partial \theta_i} = 0$$

$$\dot{\theta}_i = \frac{\partial H}{\partial J_i} = \Omega(\mathbf{J})$$



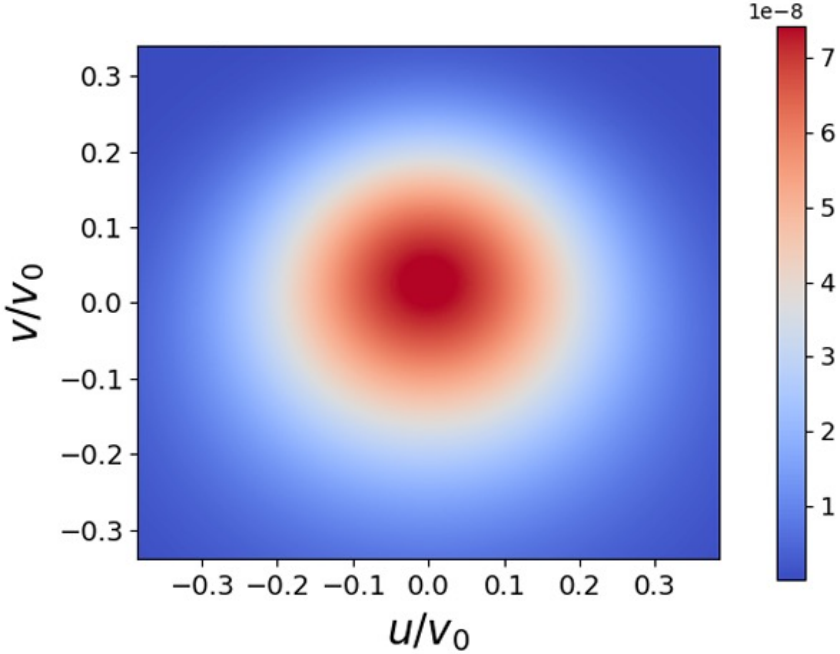
- Natural phase-space coordinates for regular orbits in (quasi)-integrable systems: **actions and angles**

=> f_0 (**J**) with **J** adiabatic invariants

Modelling the MW disk

Adjust combination of parametric DFs (e.g. Binney & McMillan 2011) :

$$f_0(J_R, J_\phi, J_z) = \frac{\Omega(R_g(J_\phi))}{(2\pi)^{3/2} 2\kappa(R_g(J_\phi))} \underbrace{\frac{\tilde{\Sigma}(R_g(J_\phi))}{\tilde{\sigma}_r^2(R_g(J_\phi))\tilde{\sigma}_z^2(R_g(J_\phi)) z_0}}_{\text{radial distribution in } R_g(J_\phi)} \times \underbrace{e^{-\frac{J_R \kappa}{\tilde{\sigma}_r^2} - \frac{J_z \nu}{\tilde{\sigma}_z^2}}}_{\text{velocity ellipsoid together with the velocity disp.dependence in previous factor}}$$



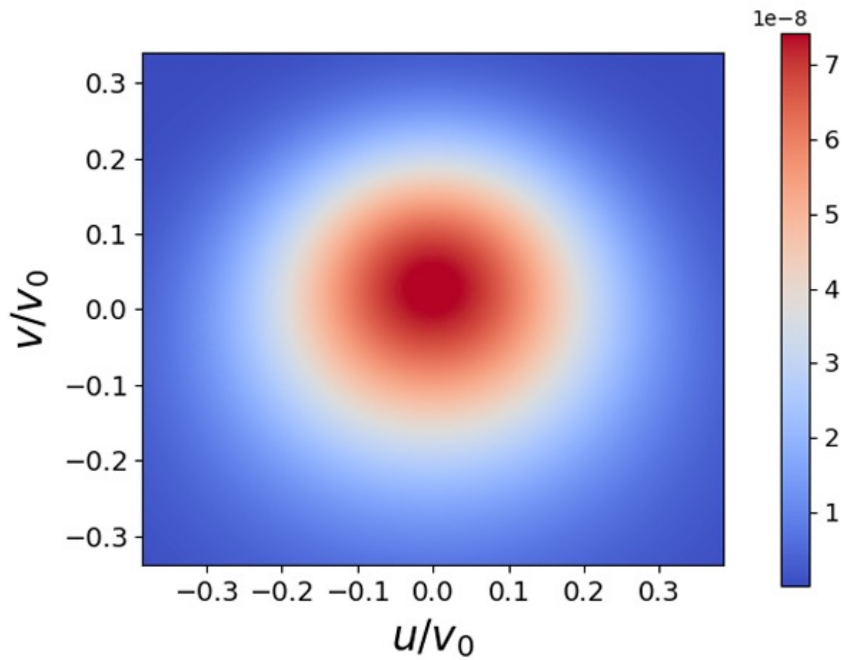
Al Kazwini et al. (2022)

Modelling the MW disk

Even better: **non-parametric DF:** adjust with neural nets + construct extended DFs, e.g.,

$f_0(\mathbf{J}) = \int \mathrm{d}Z\, F_z\, (Z,\mathbf{J})$ (+ *alpha-elements*, *ages*,…)

But not so « simple »: disk perturbed by both internal non-axisymmetries and external perturbations!



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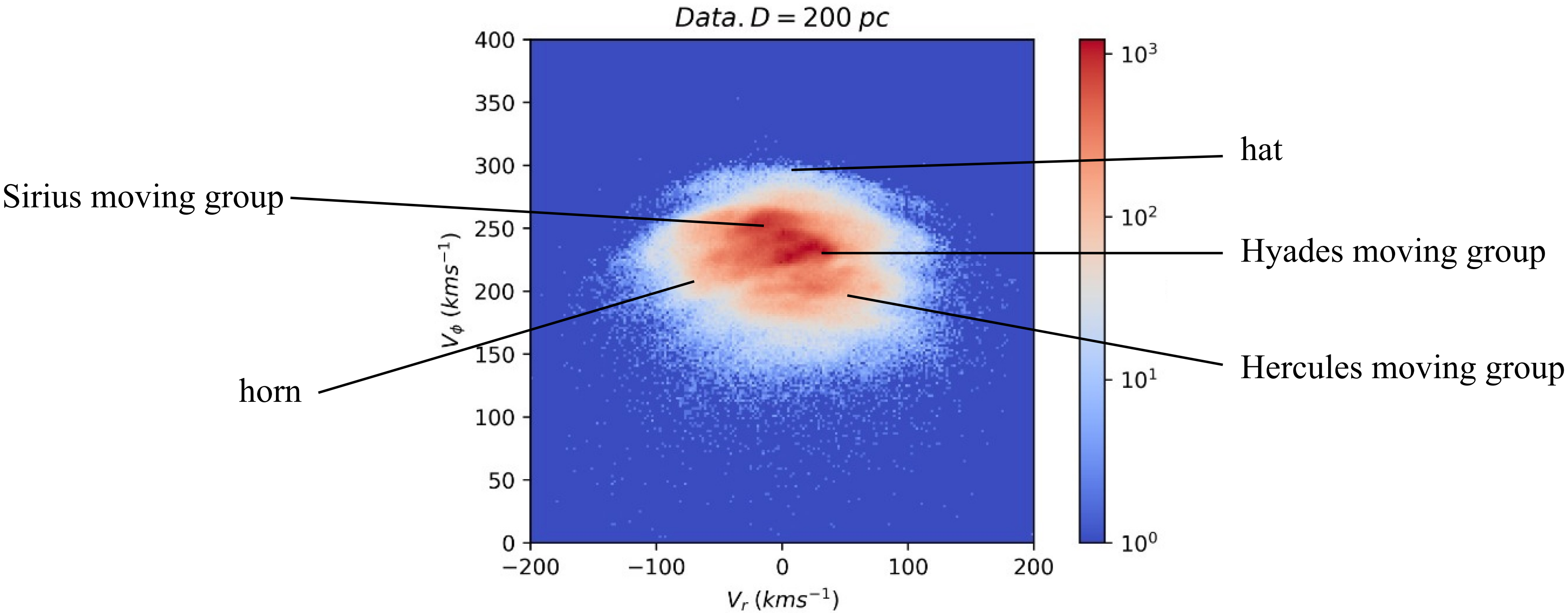

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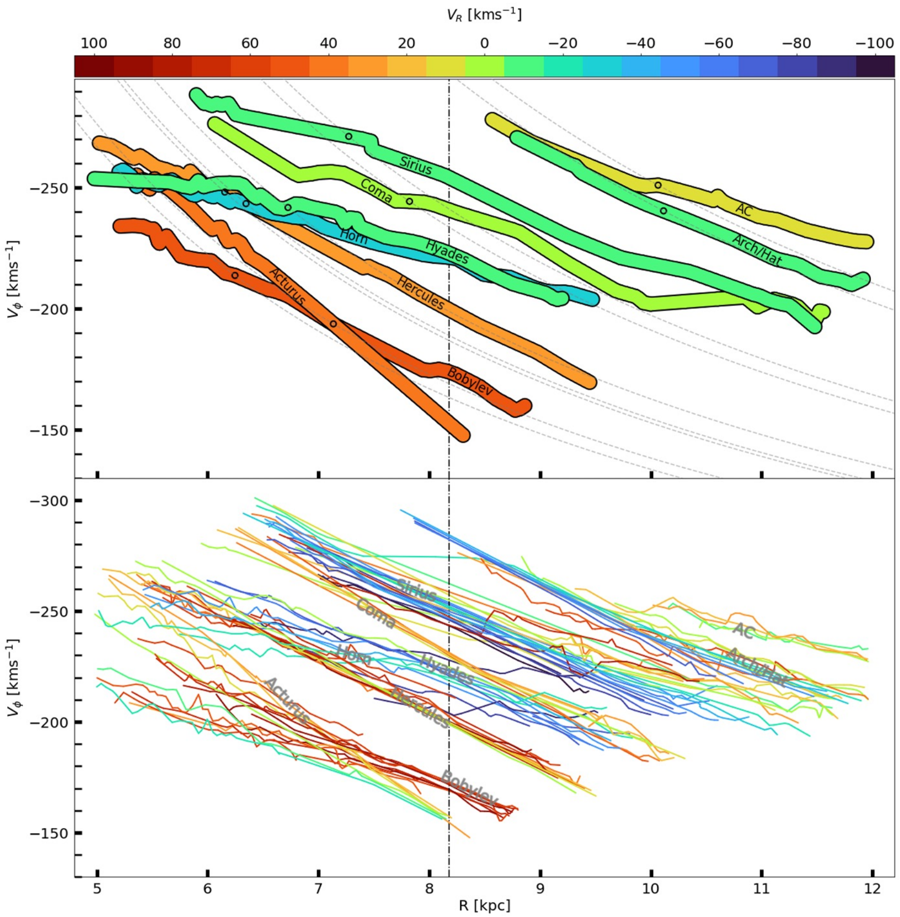
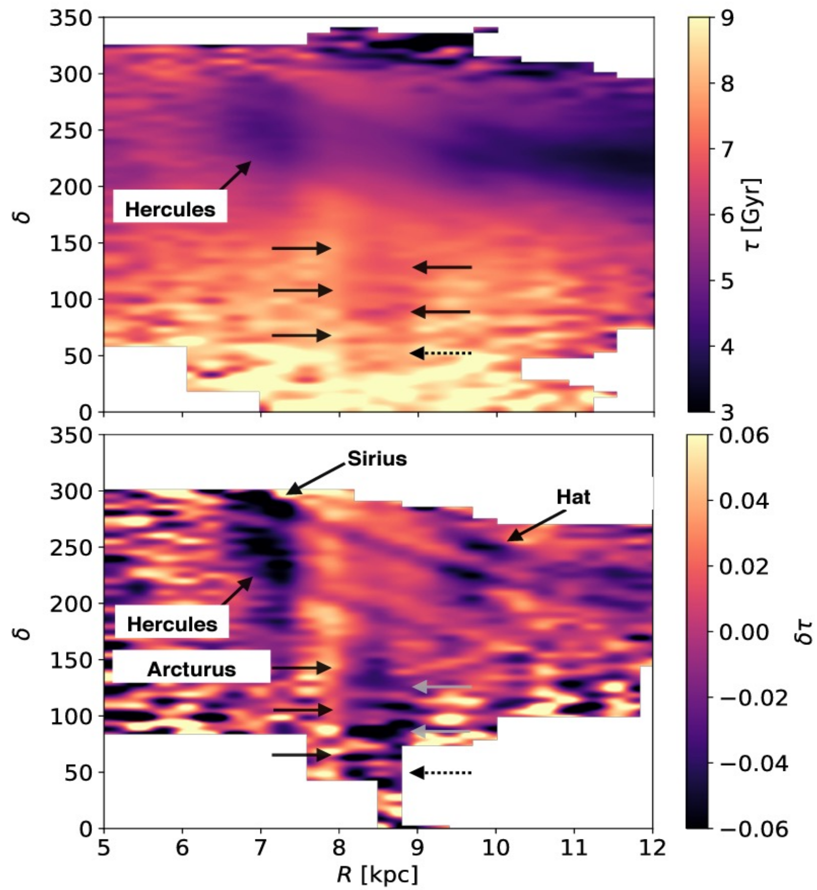


Bernet et al. (2022), EDR3



Laporte et al. (2020), DR2

Adding a bar and spiral arms

$$\boxed{\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\partial f_0}{\partial \mathbf{J}} \cdot \frac{\partial \Phi_1}{\partial \boldsymbol{\theta}}}$$ LCBE

$$\Phi_1(R,\varphi,z) = \mathrm{Re}\left\{\sum_{j,l} \phi_{jml}(J_R, J_z, J_\varphi)\mathrm{e}^{\mathrm{i}(j\theta_R + m\theta_\varphi + l\theta_z)}\right\}$$

Integrate from zero amplitude bar to plateau of constant amplitude:

$$f_1(\boldsymbol{J},\boldsymbol{\theta},t) = \mathrm{Re}\left\{\sum_{j,l=-n}^{n} f_{jml}\,\mathrm{e}^{\mathrm{i}[j\theta_R + m(\theta_\varphi - \Omega_\mathrm{p}t) + l\theta_z]}\right\},$$

$$f_{jml} = \phi_{jml} \times \frac{j\frac{\partial f_0}{\partial J_R} + m\frac{\partial f_0}{\partial J_\varphi} + l\frac{\partial f_0}{\partial J_z}}{j\omega_R + m(\omega_\varphi - \Omega_\mathrm{p}) + l\omega_z}$$

Monari et al. (2016); Al Kazwini et al. (2022)

Resonances

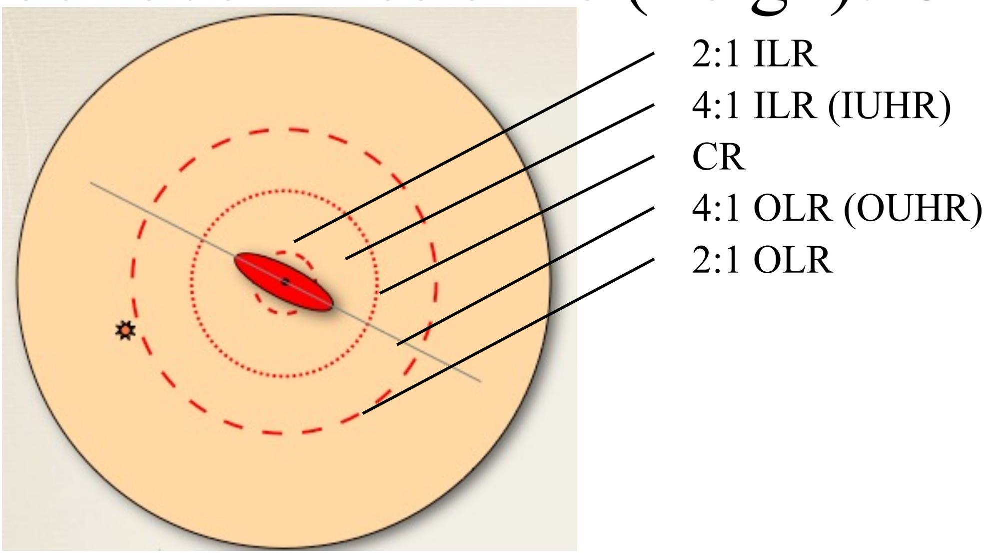
$m[\omega_\phi(R) - \Omega_p] = +- \, \omega_R(R)$ **<=> m:1 resonance**

Lindblad resonance in case of **m-fold symmetry**

=> m radial oscillations while doing 1 rotation in perturber frame

If stars overtake the perturber (+ sign): ILR

If perturber overtakes stars (- sign): OLR



Backward integration

At resonances, make two successive canonical transformations, separate slow and fast variables, average over fast ones, and compute action-angle variables of the slow ones (Monari et al. 2017)

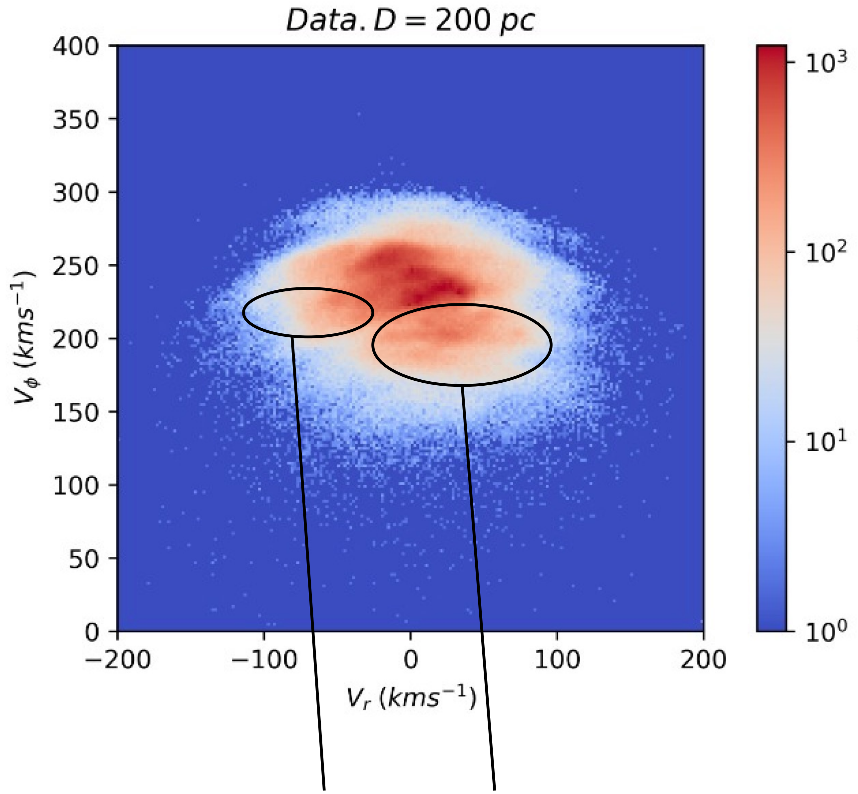
BUT complicated with multiple patterns (resonance overlaps, Minchev & Famaey 2010)

=> backward integrations: conservation of the DF in infinitesimal phase-space patches following the Hamiltonian flow, which allows us to compute the current DF by integrating orbits backward in time to an axisymmetric equilibrium state, where analytical expression for the DF

$$\boxed{f_{\mathrm{T}}(p_1, t_1) = f_{\mathrm{T}}[p(t_0), t_0]}$$

Vauterin & Dejonghe (1997)

Hercules-Horn

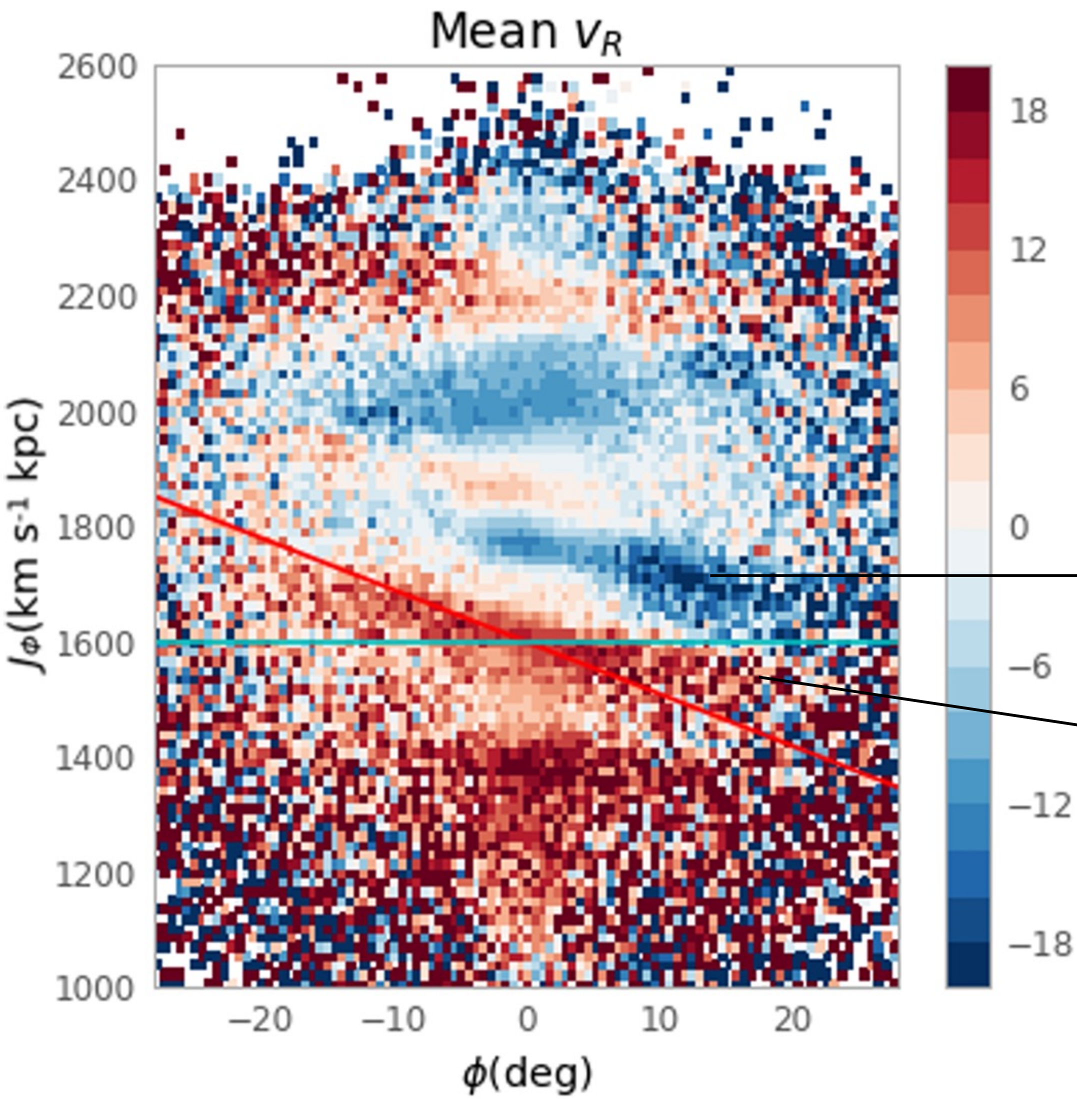


Hercules+Horn, characteristic of the **OLR**
=> $\mathbf{\Omega_b \sim 50\text{-}55}$ **km/s/kpc**

Other possibility : Hercules **corotation**,
horn 6:1 OLR of the m=6 mode of the bar
=> $\mathbf{\Omega_b \sim 35\text{-}40}$ **km/s/kpc**

Bar pattern speed

Gaia DR2, annulus of 0.4 kpc around the Sun's Galactocentric radius



Horn, no variation with azimuth for OLR

Hercules, varies with azimuth for CR

Monari, Famaey, et al (2019) => « Slow » bar (35-40 km/s/kpc)

Bar pattern speed

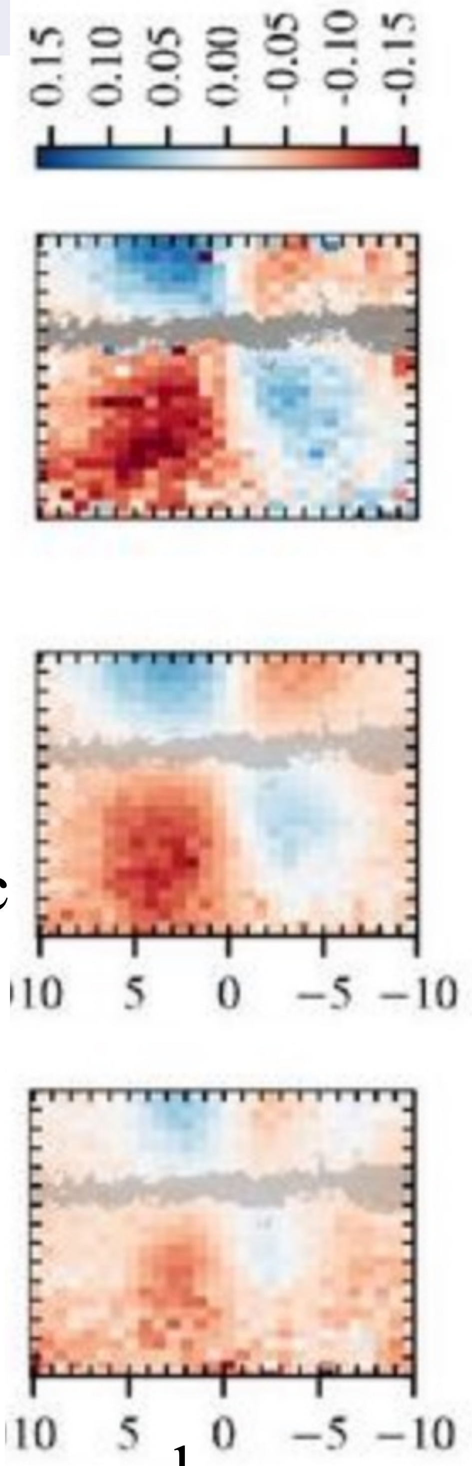
1.75×10^{8} PMs at
$-10 < l < 10^\circ$,
$-10^\circ < b < 5^\circ$
in the VVV Infrared Astrometric Catalogue (VIRAC), calibrated on Gaia DR2 (Clarke et al. 2019)

$\frac{\sigma_{lb}^2}{\sigma_l \sigma_b}$ obs.

b

37.5 km/s/kpc

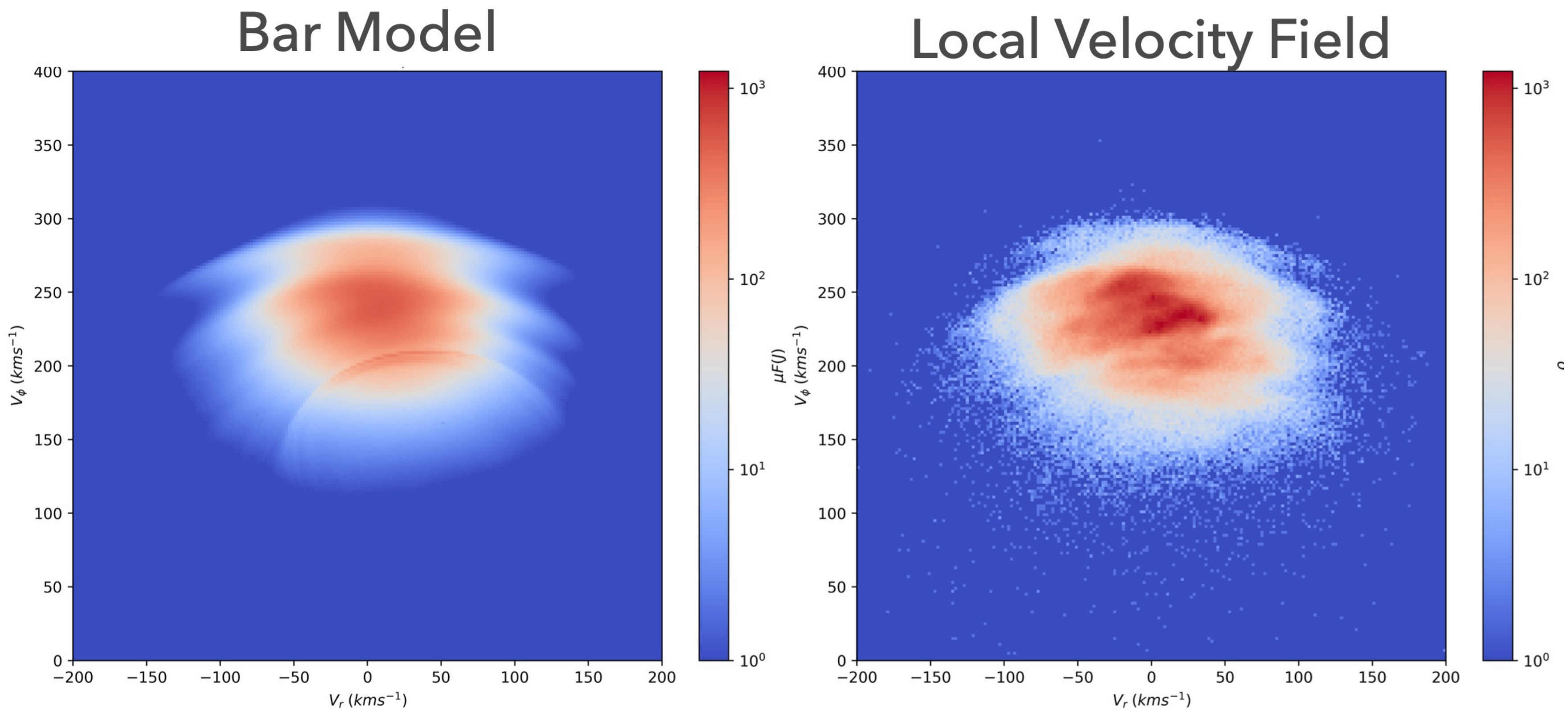
50 km/s/kpc



l

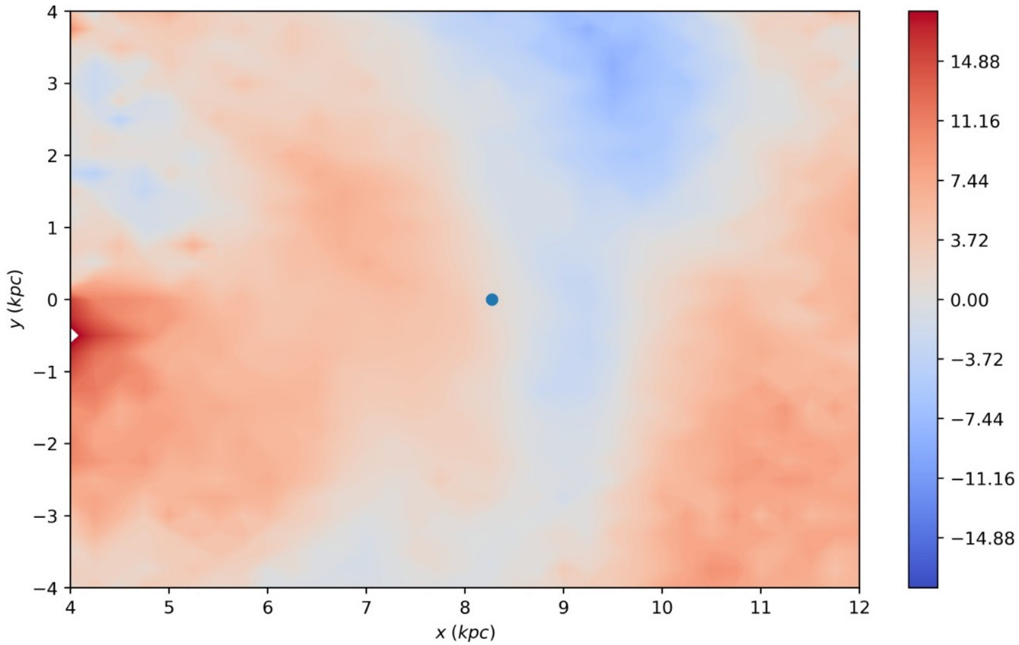
See also Sanders et al. (2019)

Backward integrations to populate the resonant zones

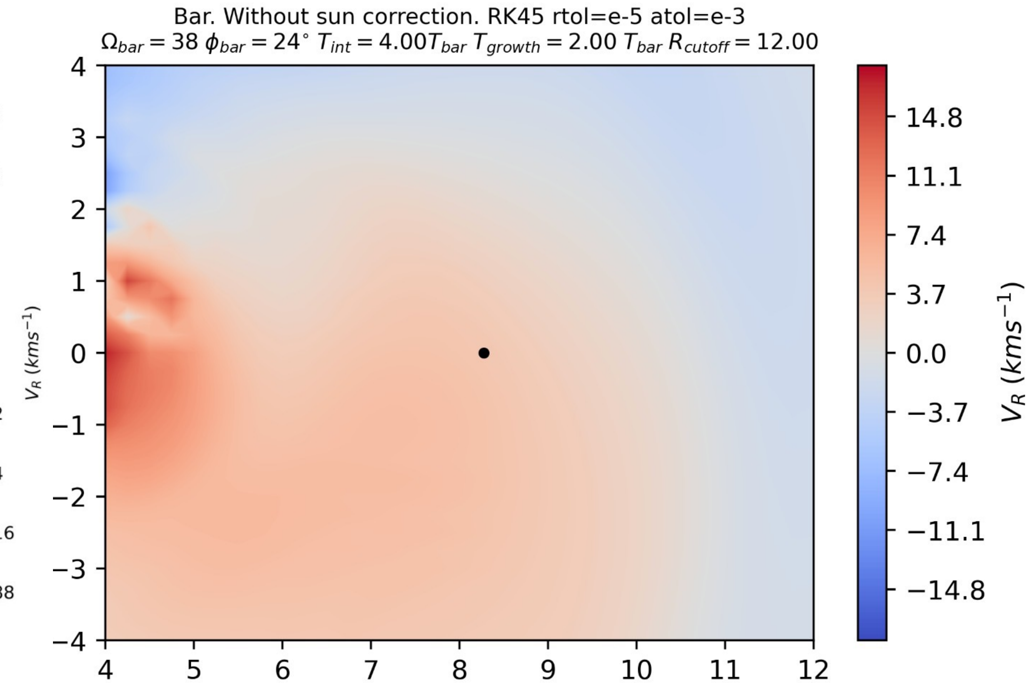


PhD thesis of Yassin Khalil (still work to do to decipher the influence of **spiral arms** and possible **time-variation of the bar pattern speed**)

Median radial velocity field

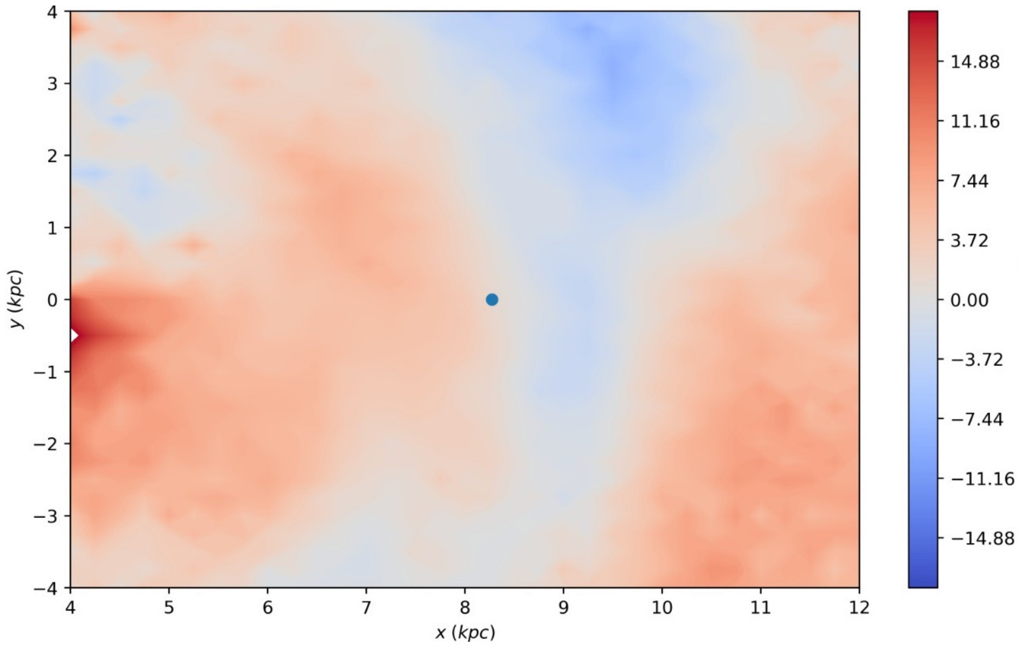


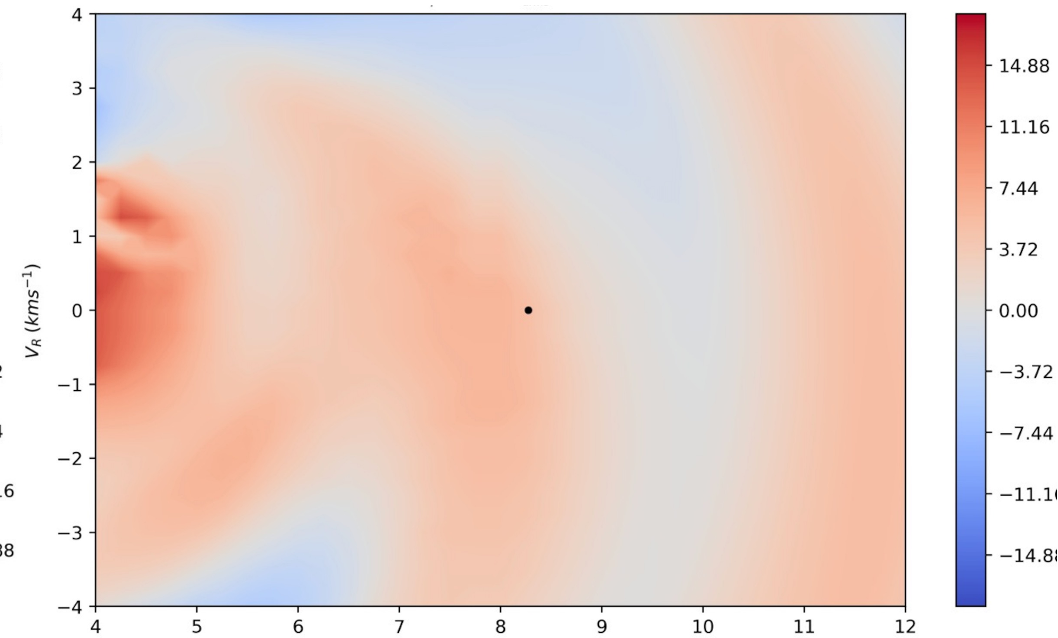
~1.3 x 10^7 stars from Gaia DR3 RVS and StarHorse distances



Bar-alone not enough (and needs small decrease of angle from 28°)

Adding spirals





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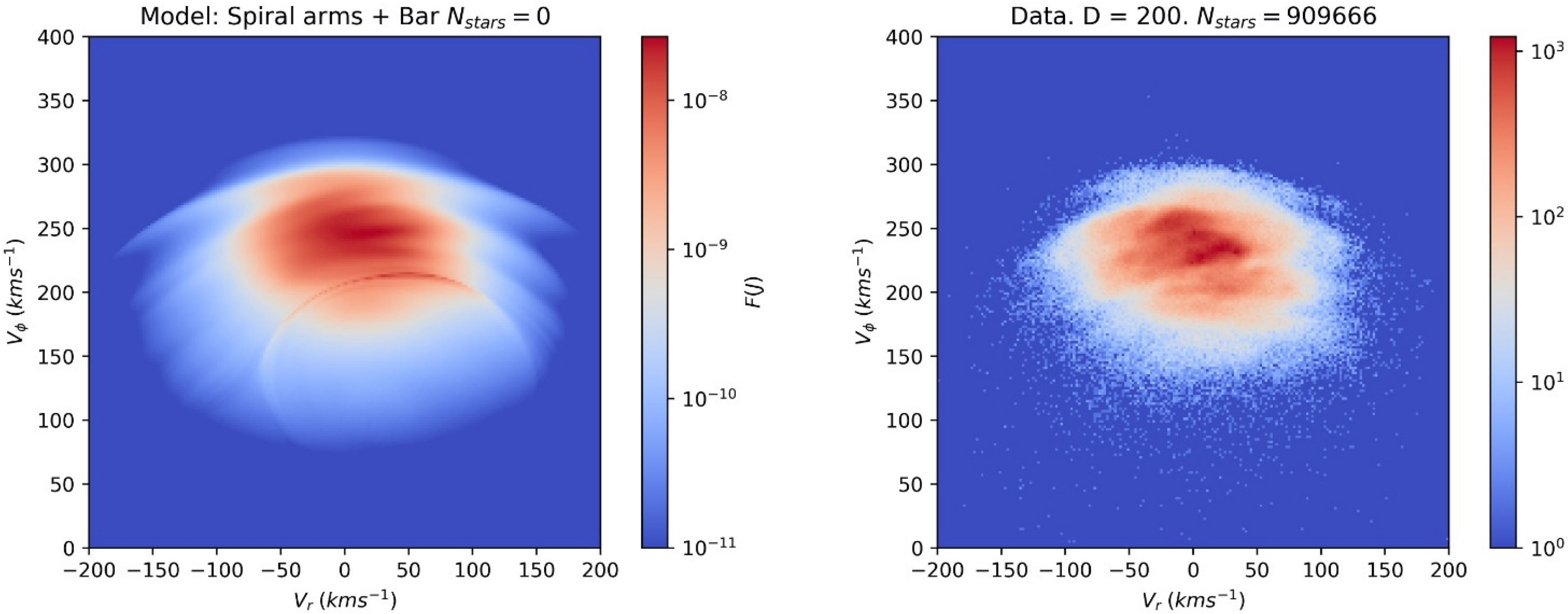
Getting there…
(here bar angle = 28°)

Scutum-Crux + Perseus/Local : not co-rotating at the Sun !

Adding spirals

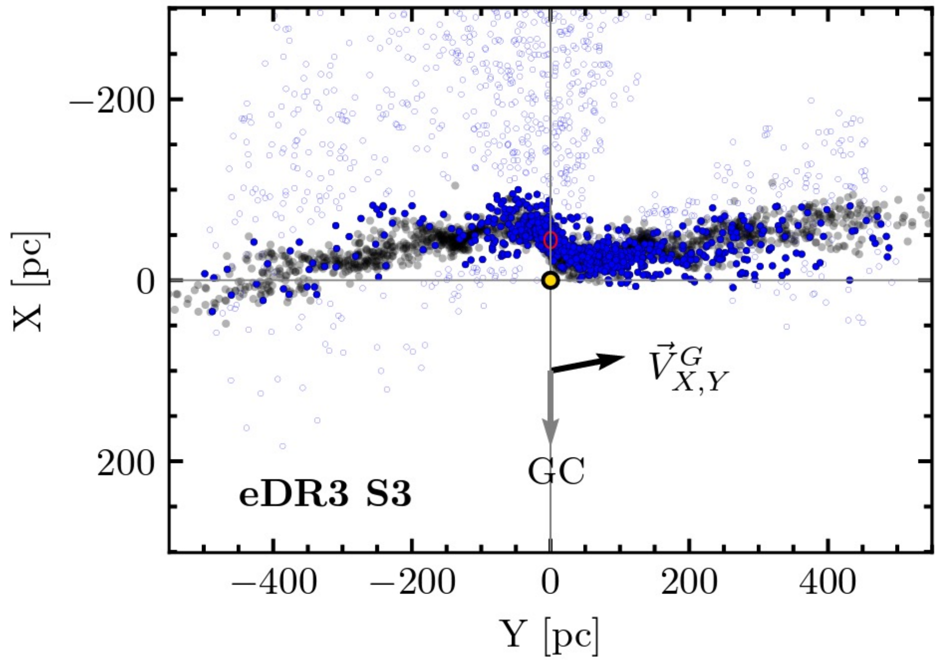


Khalil et al. in prep.

+ slow-down of the bar ?

Disk tidal streams: a new probe

With Gaia, tidal tails of open clusters in the disk have started being discovered (combination of exquisite Gaia data and detailed N-body simulations)



Jerabkova et al. (2021)

The bar exerts torques on orbits

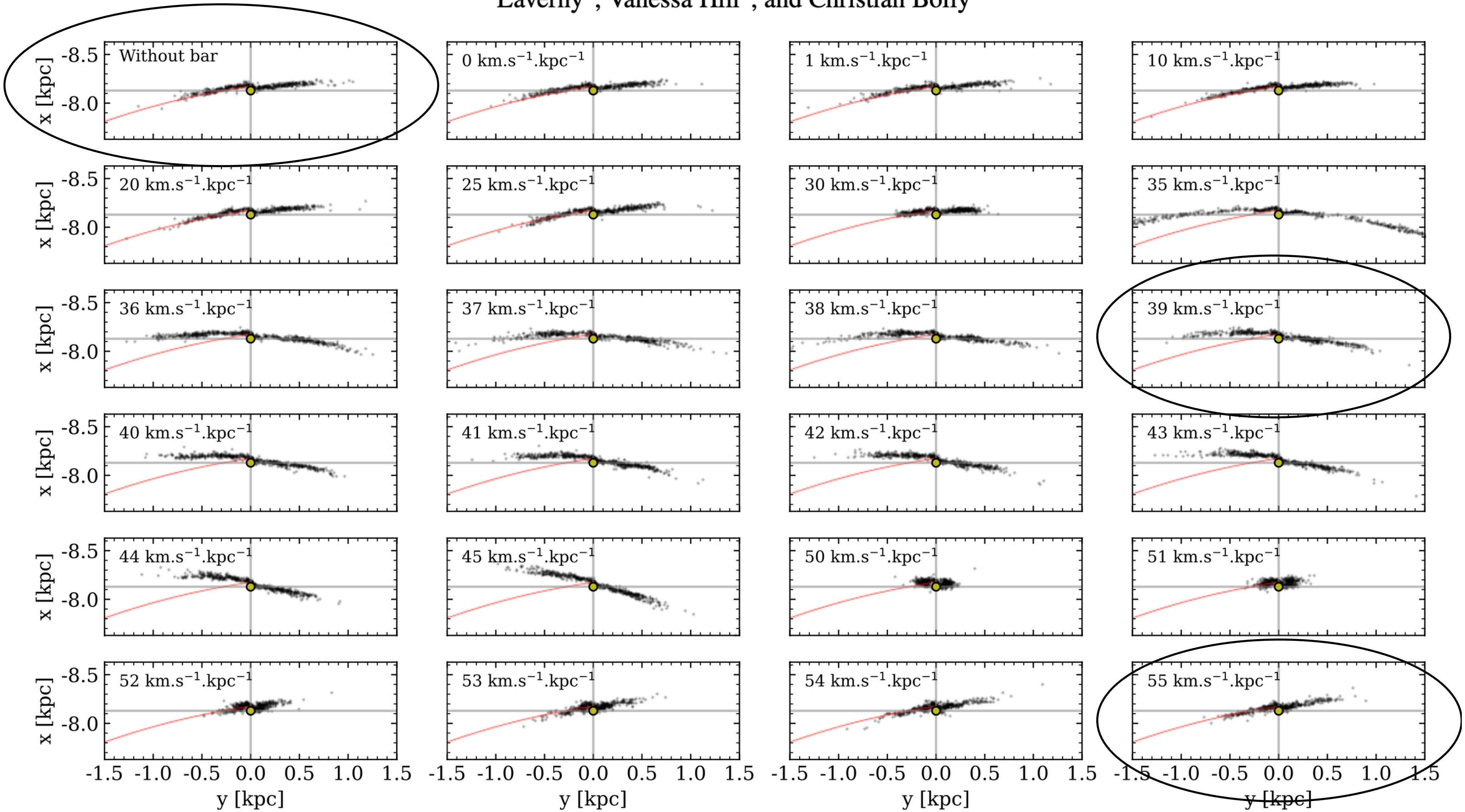
- $L_z = J_\phi$ conserved in axisymmetric potentials but not in a barred one
- Oscillation especially important at resonances (remember that J_ϕ then oscillates as a pendulum)
- Because of conservation of Jacobi integral $E_J = E - \Omega_b J_\phi$, variations of J_ϕ also imply variations of energy

⇒ ***« shepherding » of streams*** (Hattori et al. 2016) : depending on the phase of the orbit, the amount of angular momentum and energy variations is different

⇒ differential changes imply different orientations (through differential angular momentum changes) and spread (through differential energy changes) of the streams

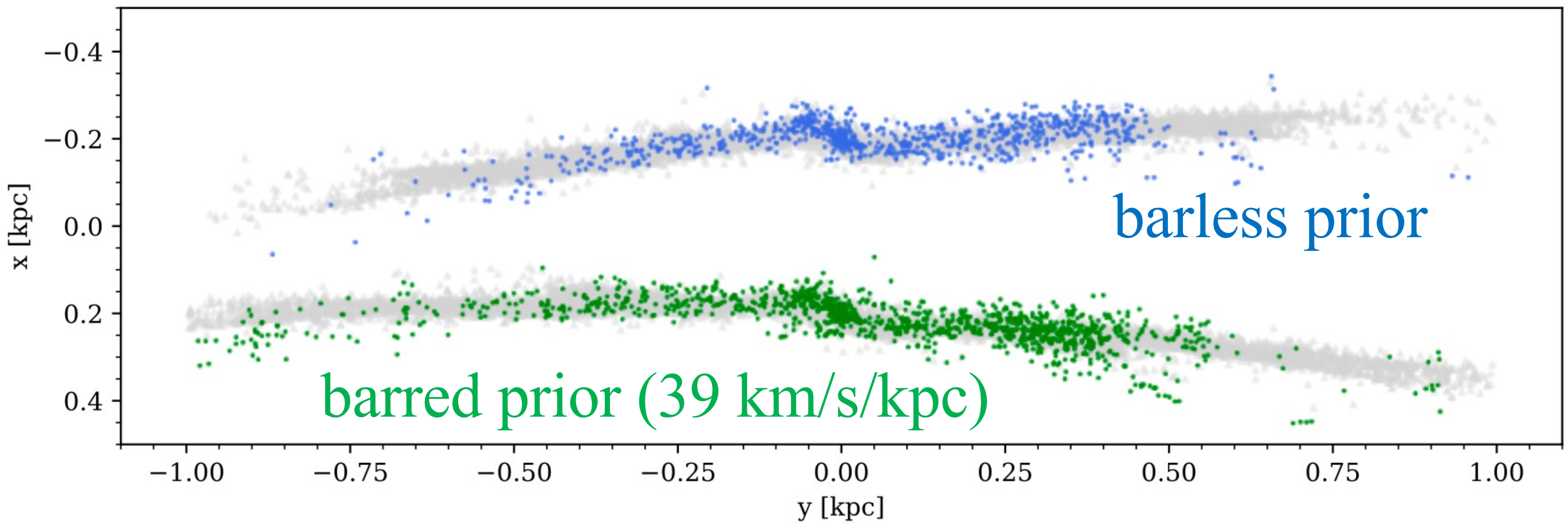
Shepherding the Hyades stream

Guillaume F. Thomas[1,2], Benoit Famaey[3], Giacomo Monari[3], Chervin F. P. Laporte[4,5,6], Rodrigo Ibata[3], Patrick de Laverny[7], Vanessa Hill[7], and Christian Boily[3]



Shepherding the Hyades stream

Bayesian membership selection from photometric filtering + kinematics

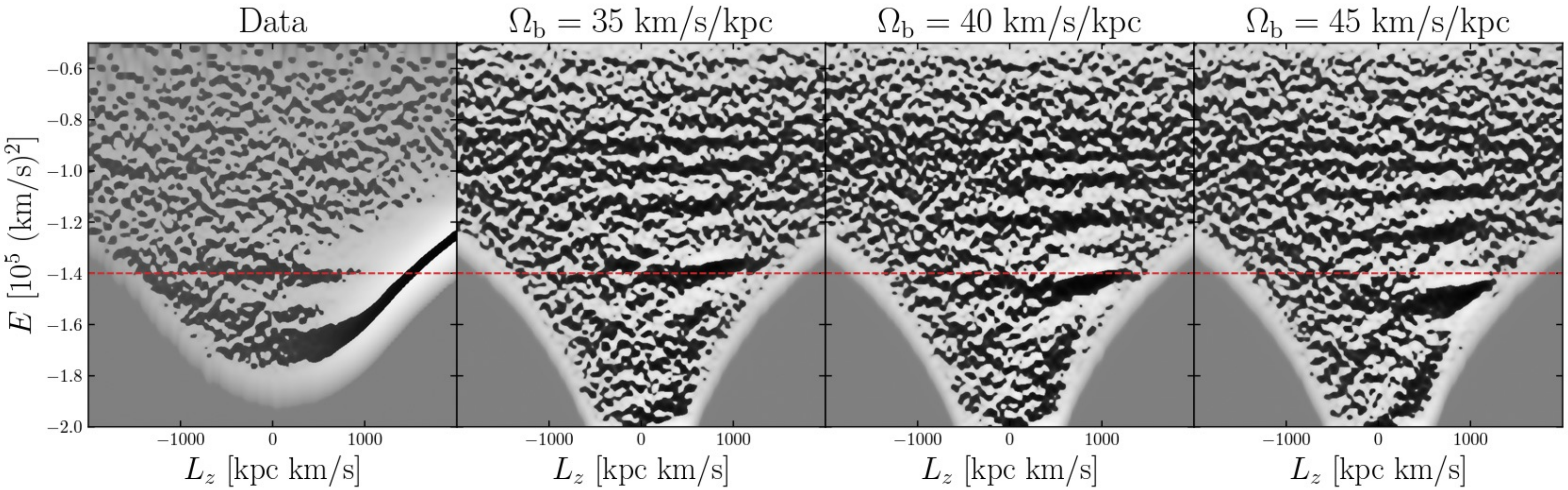


- Both selections well populated => stars from the disc having similar photometry/dynamical properties as Hyades > number of stars from stream itself

- Needs to add spirals

- Needs HR chemical labelling ! (Li at 5000 $K < T_{eff} < 6500$ K)

The bar affects the stellar halo too

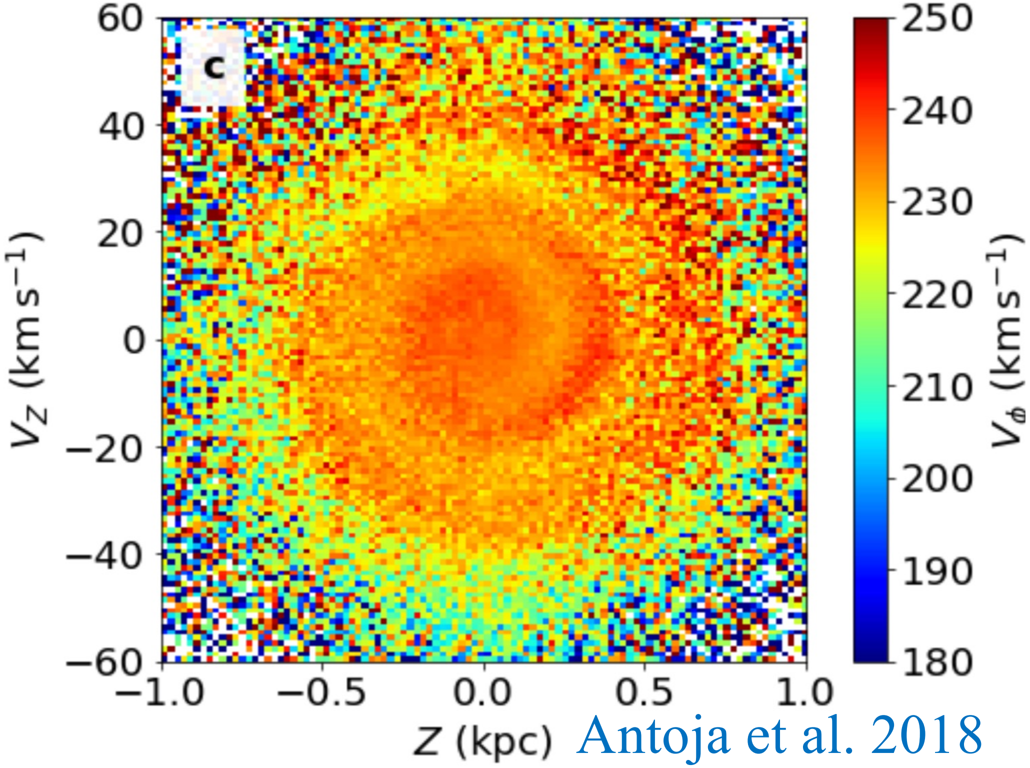


Dillamore et al. (2023)

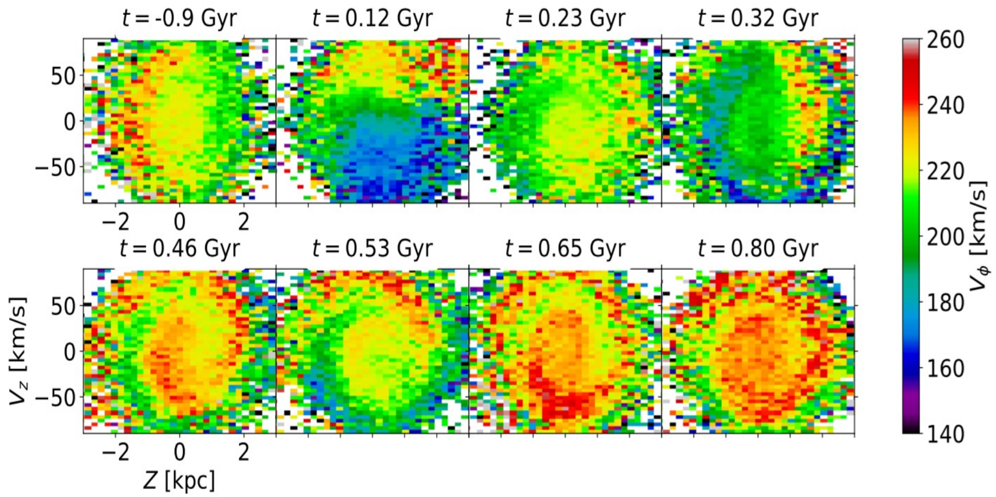
~25% of UMP stars with [Fe/H] < -4 are on prograde orbits confined within 3 kpc of the Milky Way plane, with J_z < 100 kpc km/s
(Sestito et al. 2019)

=> effect of the bar on the pre-historic hot Milky Way "Aurora" ?

The vertical kinematics of the disk



Antoja et al. 2018



Laporte et al. 2018
(last pericentric passage of Sgr dwarf at t=0)

Can traditional Jeans modelling be applied? **NO** (Haines et al. 2019)

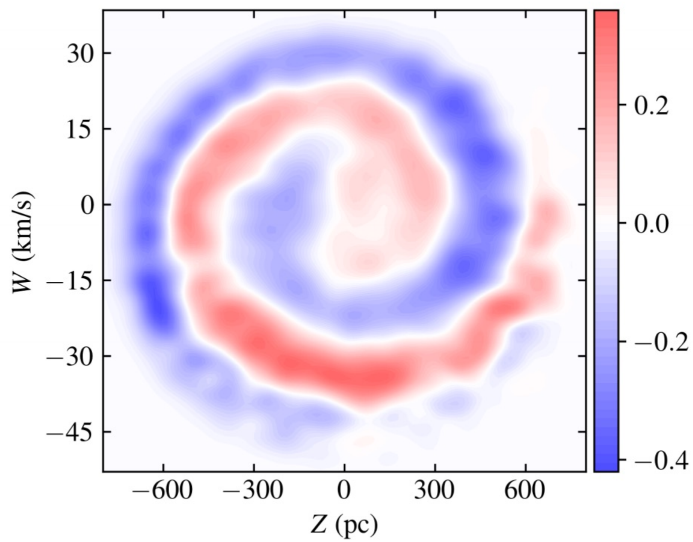
One-armed snail from the Sgr dwarf impact and halo deformation

Local dark matter density ?

Non-equilibrium => needs development of appropriate framework including self gravity in 3D

But… **first attempts**, in 1D and neglecting self-gravity (Binney & Schonrich 2018; Widmark et al. 2021)

Perturb $f(J_z)$ into $f(J_z, \theta_z)$ and let each star oscillate with its **own vertical frequency** which depends on the **Hamiltonian**
$\Rightarrow$Shape of phase-spiral depends on the potential and time since pert.

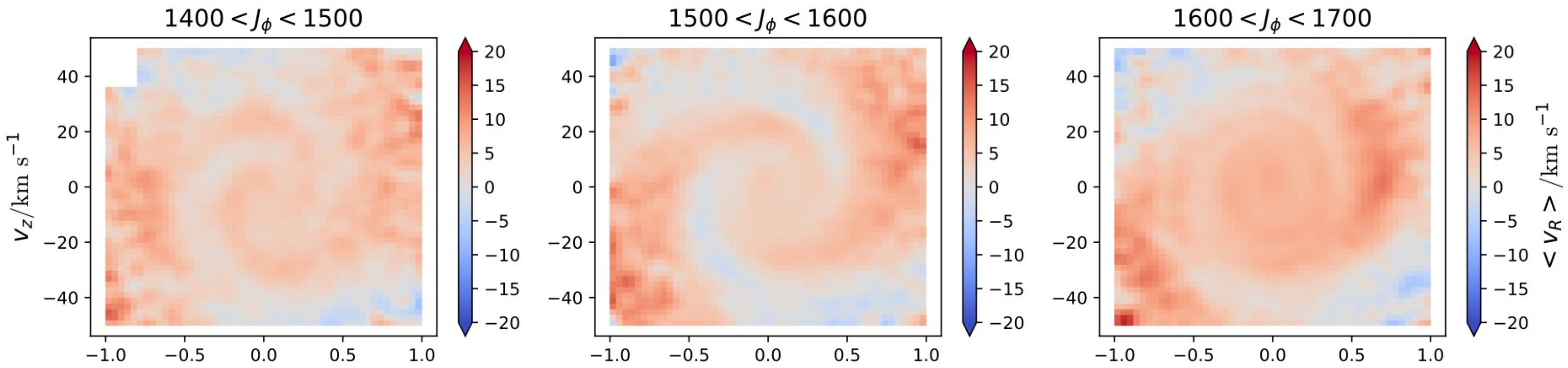


Widmark et al. (2021) fit to quasi-circular orbits, compare potential to baryonic one and infer

$\rho_{DM} = 0.0085 \pm 0.004\ M_\odot/pc^3$

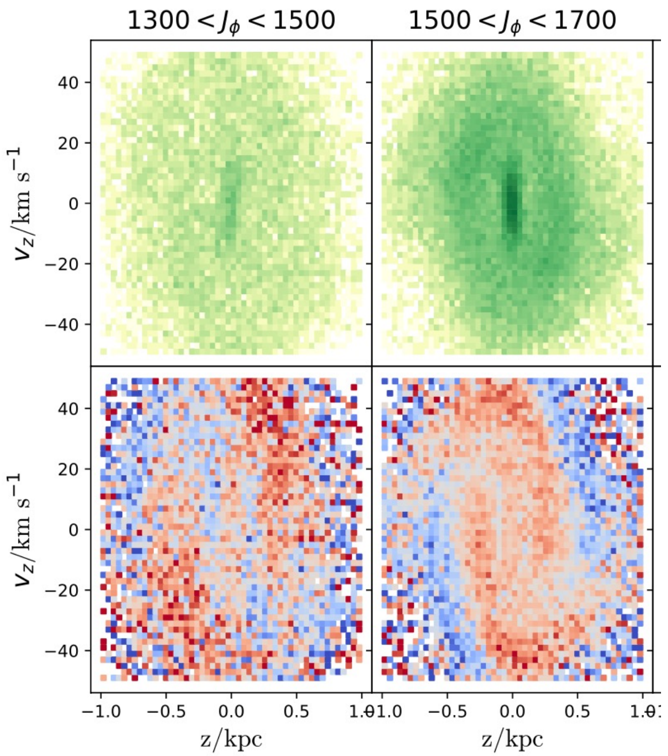
$= 0.32 \pm 0.15\ GeV/cm^3$

But the plot thickens…



Li et al. (2023) Gaia DR3 RVS

Two-armed phase spiral!

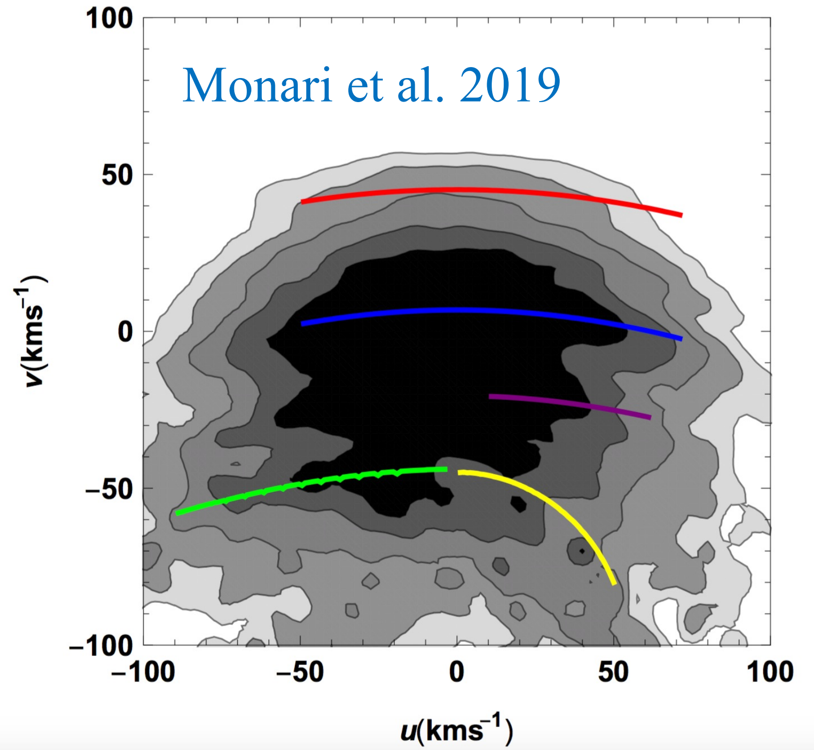


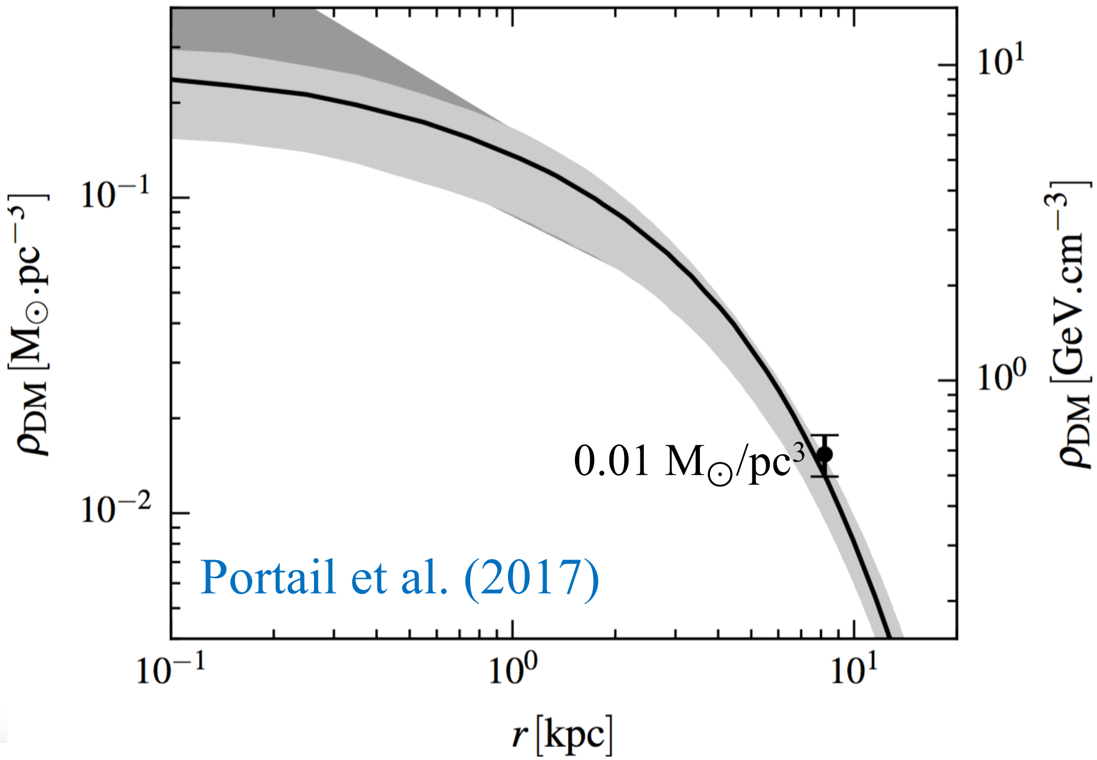
<= decelerating bar toy-model (no Sgr !)

(a decelerating bar might indicate an interaction with the dark matter halo)

A dark matter core in the MW?

- Bulge mass (2.2 kpc, 1.4 kpc, 1.2 kpc): $1.85 \times 10^{10}\, M_\odot$
  - Dark matter mass: $3.2 \times 10^{9}\, M_\odot$





Bar model + keep the RC constant between 6 kpc and 8 kpc
=> **cored dark matter profile at the center !?**

Perspectives:

-Improve our out-of-equilibrium dynamical analyses !

-While waiting for DR4 and DR5 …

TO BE CONTINUED !